Hydraulics of Wetted-wall Columns

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On the assumption of laminar Newtonian flow, flow profiles are developed for both the vertical plate and cylindrical column by use of classical equations. From the flow profiles the mass transport, vertical pressure gradient, and optimal design parameters are calculated. With the Reynolds-number criterion used to ascertain the maximum nonturbulent velocity, minimal values for the plate distance and column radius can be calculated. An example is included for each case.

Where laminar Newtonian flow prevails, the flow profiles of both liquid and vapor streams can be computed by elementary classical methods. From these profiles one can find all other hydraulic characteristics of flow. The following development considers the wetted-wall column for two cases, the vertical flat plate and the cylindrical column, where it is assumed that laminar Newtonian flow prevails.

The development in each of the two cases can be broken up into three areas of interest. The first two sections in both cases deal with obtaining the differential equation in terms of velocity from equilibrium of forces and solving this differential equation by use of boundary conditions. These sections may or may not be of interest to most engineers. The third section in both cases deals with concepts of mass transport and pressure gradient in wetted-wall columns, which are obtained directly from the velocity equations. The developments in the three sections can stand alone.

The fourth section introduces the use of the Reynolds number of the vapor to determine a so-called "critical plate distance," or radius. The expression for velocity in the vapor stream as predicted by the Reynolds number contains the vapor-stream width, or diameter (hydraulic diameter), and if one assumes that the average velocity over the vapor-stream width is to remain less than that predicted by the use of Reynolds number, a simple mathematical manipulation will yield an upper boundary on plate distance, or radius, for any fixed ratio of vapor-stream width to plate width, or radius. Only the upper bound is of interest here, since for any fixed ratio a decrease in the plate width or radius would lessen both total mass flow and mass flow per unit area. The lower limit of plate distance or radius based upon vapor transport per unit area for *all* ratio values from 1 to 0 is of value, as it forms a lower boundary for design purposes. This expression is developed, and two examples showing the use of the expressions developed are included. A three-dimensional graph is also included to show what the author has accomplished with the foregoing development.

It is hoped that the results of the fourth section can be used to interpret flow in packed columns after experimental verification, as it would otherwise be of no value. This might be accomplished by considering the packed column as an aggregate of small individual cylinders or large concentric cylinders.

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VERTICAL FLAT PLATES

Basic Equations

The column is bounded by two vertical parallel planes separated by the distance $2x_0$. The reflux liquid flows downward by gravity as film on the walls and is symmetrically divided between them. Cartesian coordinates are taken with the yz plane parallel to and midway between the walls with the z axis vertically upward. The linear velocity of flow upward is W and horizontal components are zero. Further, for a given mass flow W is a function of x alone. The two interfaces are the planes $x = \pm x_s$, where x_s, the half thickness of the vapor path, is to be determined from the given operating conditions.

The fluids are both assumed to be Newtonian; that is, the tractive force per unit area is assumed to be equal to the product of the viscosity η, a constant property of the material, and the shear rate dW/dx. This law provides the starting point for the derivation of the flow distribution.

An element of volume in the liquid has width Δx, a unit length in y direction, and a thickness Δz in z direction. Steady state conditions in the liquid are defined as

$$F_1 + F_2 + F_3 = 0$$

where

F = gravitational force = $\Delta x \cdot \Delta z \cdot 1 \cdot \rho_l g$
F_2 = pressure gradient force = $(\Delta p/\Delta z) \cdot \Delta z \cdot \Delta x \cdot 1$
F_3 = shear force = ΔF_x = $\eta_l (\Delta/\Delta x)(dW/dx)\Delta z \cdot \Delta x \cdot 1$

Simplifying and passing to the limit, one gets the basic differential equation

$$\eta_l \frac{d^2 W_l}{dx^2} + Q = 0 \qquad (1)$$

where $Q = (dp/dz) + \rho_l g$

The corresponding vapor equation can be written as

$$\eta_v \frac{d^2 W_v}{dx^2} + \frac{dp}{dz} = 0 \qquad (2)$$

since density term can be neglected.

If it is assumed that pressure gradient is only a function of height z (dp/dz is independent of x), integration of Equation (1) twice yields

$$\eta_l W_l + \frac{x^2}{2} Q = C_1 x + C_2 \qquad (3)$$

Integrating Equation (2) once gives

$$\eta_v \frac{dW_v}{dx} + x \frac{dp}{dz} = C_3 \qquad (4)$$

(dp/dz is independent of x)

and another integration yields

$$\eta_v W_v + \frac{x^2}{2} \frac{dp}{dz} = C_3 x + C_4 \qquad (5)$$

Boundary Conditions

In the vapor stream $dW_v/dx = 0$ at $x = 0$; hence from Equation (4) $C_3 = 0$, and in the liquid stream, since a no-slip condition at the walls is assumed,

$$W_l = 0 \text{ for } x = x_0 \text{ and } x = -x_0$$

Equation (3) therefore becomes

$$\eta_l W_l = \frac{Q}{2}(x_0^2 - x^2) \qquad (6)$$

When a no-slip assumption is made at the interface, the limiting liquid and vapor velocities on the respective opposite sides of that plane are assumed equal, i.e., $W_l = W_v$ at $x = x_s$, and one gets upon substituting from Equation (5) with

$C_3 = 0$ into Equation (6) a value for C_4 so that finally Equation (5) may be written as

$$\eta_v W_v = \frac{1}{2}\left[\frac{\eta_v}{\eta_l} Q\,(x_0^2 - x_s^2) + \frac{dp}{dz}(x_s^2 - x^2)\right] \quad (7)$$

This completes the calculation of the flow profiles, Equation (6) expressing the velocity of the liquid in any plane x where $x_s \leq x \leq x_0$ and Equation (7) giving the vapor velocity for x where $0 \leq x \leq x_s$.

Expressions for dp/dz and Mass Transport

The velocities W_v and W_l calculated for the liquid and vapor streams are point functions and consequently are inherently dependent variables of only indirect engineering importance. The total mass flow rates, however, are of primary importance and can be calculated from the velocities by integrating them over the cross-sectional area of the streams. Thus

$$M_l = 2\rho_l \int_{x_s}^{x_0} W_l\, dx = \frac{\rho_l}{3\eta_l} Q(2x_0^3 - 3x_s x_0^2 + x_s^3) \quad (8)$$

and

$$M_v = 2\rho_v \int_0^{x_s} W_v\, dx = \frac{2}{3}\frac{\rho_v}{\eta_v} x_s^3 \frac{dp}{dz} + \frac{\rho_v}{\eta_l} Q(x_0^2 x_s - x_s^3) \quad (9)$$

In practice the total mass flow rates are usually regarded as related to each other by an independent parameter, the reflux ratio R, where

$$R = (M_v - M_l)/M_v$$

The liquid holdup in the unit-area column is given by

$$H_l = \rho_l(x_0 - x_s)$$

Unfortunately, this is not uniquely determined for x_s and x_0 independently, as it depends on the ratio x_s/x_0. Use of the Reynolds number to determine the onset of turbulence does fix the maximum value for x_0 for any given ratio x_s/x_0 when the system just reaches the turbulent stage. That there is a minimum value for x_0 will also be shown. For the important case of $M_v = M_l$, equating Equation (9) to the negative of Equation (8) gives for the pressure gradient

$$\frac{dp}{dz} = -\rho_l g \cdot \frac{(3k_1 - 1)u^3 + 3(1 - k_1)u - 2}{(1 - 3k_1 + 2k_2)u^3 - 3(1 - k_1)u + 2} \quad (10)$$

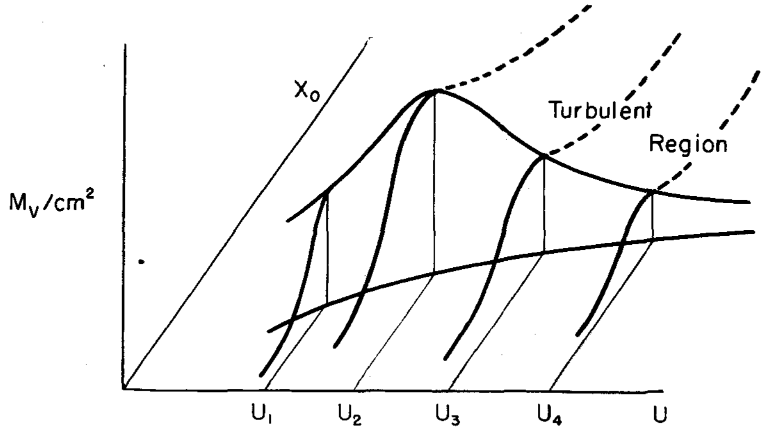

Fig. 1.

where

$u = x_s/x_0$, $k_1 = \rho_v/\rho_l$, $k_2 = k_1\eta_l/\eta_v$

This relation can be plotted graphically, and from such a curve u can be determined for values of dp/dz. $[(1/\rho_l g)(dp/dz)]$ is monotonic and is -1 when $u = 0$ and equal to zero when $u = 1$. To determine x_0 another criterion is necessary.

Equation (9), for use in the fourth section, can be written as

$$M_v = \rho_v x_0^3 u\left[\frac{2}{3\eta_v}\frac{dp}{dz}u^2 + \frac{Q}{\eta_l}(1 - u^2)\right] \quad (10a)$$

which is in gram seconds^{-1}, and the average mass transport in (gram centimeters^{-2})(second^{-1}) is

$$\frac{M_v}{2x_s} = \rho_v \frac{x_0^2}{2}\left[\frac{2}{3\eta_v}\frac{dp}{dz}u^2 + \frac{Q}{\eta_l}(1 - u^2)\right] \quad (10b)$$

where again $u = x_s/x_0$

Critical Values for Plate Distance

An expression for the average velocity in the vapor stream can be obtained from the integral of the expression for the velocity equation (7). Thus

$$\overline{W}_v = \frac{1}{x_s}\int_0^{x_s} W_v\, dx = \frac{x_s^2}{3\eta_v}\frac{dp}{dz} + \frac{Q}{2\eta_l}(x_0^2 - x_s^2) \quad (11)$$

If it is assumed that the average velocity must be less than (or equal to) the velocity determined by the Reynolds number, then $W_{Re} \geq \overline{W}_v$

where

$$W_{Re} = \frac{Re\eta_v}{D\rho_v} = \frac{Re\eta_v}{4x_s\rho_v} \geq \overline{W}_v \quad (11a)$$

since $4x_s = D$ (hydraulic diameter) and Re = Reynolds number.

Substituting expression (11) for $\overline{W}_v$ in (11a) and solving the inequality for x_0^3 one gets

$$x_0^3 \leq \frac{Re\eta_v}{2\rho_v u\left[\frac{2}{3\eta_v}\frac{dp}{dz}u^2 + \frac{Q}{\eta_l}(1 - u^2)\right]} \quad (12)$$

where again $u = x_s/x_0$.

From simple considerations of the denominator in (12), it is obvious that x_0 satisfies the inequality

$$x_0^3 \geq \frac{Re\eta_v}{2\rho_v \operatorname{Max} u\left[\frac{2}{3\eta_v}\frac{dp}{dz}u^2 + \frac{Q}{\eta_l}(1 - u^2)\right]} \quad (13)$$

This denominator in (13) has a maximum, as it is 0 for both $u = 0$ and $u = 1$ since $(dp/dz) = 0$ when $u = 1$. It is positive throughout the range $0 \leq u \leq 1$.

To interpret the inequalities (12) and (13) an expression for mass vapor transport is required. This has been developed as Equation (10a). If the equality for x_0^3 in Equation (12) is substituted in (10a), then the mass vapor transport in gram-seconds^{-1} at the average critical velocity becomes

$$M_v = \frac{Re\eta_v}{2} \quad (14)$$

The inequality (12) states that for a fixed u, x_0 cannot exceed the right member as the critical velocity would be exceeded, and Equation (10a) clearly states that the mass vapor transport would decrease with decreasing x_0 and fixed u. The inequality (13) forms an absolute lower bound for x_0 for all u (Figure 1).

The following example illustrates the development. Values of u were arbitrarily chosen and the values of the pressure gradient from Equation (10) were calculated as shown in the second column in Table 1. To show that the denominator of the right member of (13) has a maximum, the third column of Table 1 was calculated by use of the results of columns 1 and 2 in the same table.

Table 2 was then readily obtained from Equations (12) and (14) and the results in Table 1. The last column in Table 2 is merely the third column divided by $2x_s$ sq. cm., as an area element of one unit in depth and $2x_s$ units wide is assumed.

Since all the calculations are based upon operating just at the critical velocity range, the interpretation of the results in Table 2 is as follows. If, for example, x_0 is chosen larger than 0.06757 cm., the value of u must be larger than 0.95 if turbulence is to be avoided. Or, if x_0 is fixed at 0.06757 cm., then any increase in the liquid layer (decreasing u) would cause the system to exceed the average critical velocity and enter the turbulent stage, since the table shows that a decreasing u must be associated with a decreasing x_0.

From an inspection of column 3 in Table 1, it is clear that the value of $u = 0.74$ and corresponding $x_0 = 0.03195$ in Table 2 form the lower bound on the half-plate distance.

Figure 1 is a three-dimensional graph of the surface with planes u = constant cutting the surface in the curves of which only the maximum ordinate is of interest in each slice. The graph is not intended to be used numerically but only as an aid to explaining the development.

Example

$\rho_l = 0.66$ g./ml. $\eta_l = 0.0027$ poise
$\rho_v = 0.0036$ g./ml. $\eta_v = 0.00011$ poise
Re of the vapor = 2,1000
$g = 980.65$ dynes/sq. cm.

TABLE 1

$u = x_s/x_0$	$1/(\rho_l g)\ dp/dz$	$1/(\rho_l g)\ M_v/x_0^3$
1	0	0
0.95	−0.03728	−0.57853
0.90	−0.14008	−2.03201
0.85	−0.29250	−3.69666
0.80	−0.45984	−4.92946
0.75	−0.61075	−5.45134
0.74	−0.63736	−5.47316
0.73	−0.66263	−5.47081
0.72	−0.68656	−5.44619

TABLE 2

$u = x_s/x_0$	x_0, cm.	M_v, g./sec.	M_v, g./(sq. cm.)(sec.)
0.95	0.06757	0.1155	0.900
0.90	0.04445	0.1155	1.444
0.85	0.03642	0.1155	1.865
0.80	0.03308	0.1155	2.182
0.75	0.03198	0.1155	2.408
0.74	0.03195	0.1155	2.442

The results of the example can be interpreted pictorially as in Figure 1. The vertical ordinate is mass transport per unit area, and the two variables in the horizontal plane are x_0 and u. The planes u_1, u_2, u_3, and u_4 cut out curves on the surface. The values for M_v/sq. cm. in Table 1 correspond to the maximum ordinates on these curves. The largest ordinate of this set of ordinates is represented on the curve with an x_0 coordinate which corresponds to minimum x_0 in the table and maximum value for M_v/sq. cm. For any fixed value of u, there is a value of x_0 which corresponds to the maximum ordinate in the slice. If in this slice (u fixed) x_0 were chosen smaller, then the M_v/sq. cm. would decrease. If the x_0 were chosen larger, the system would no longer be in laminar flow but turbulent, and hence the mass transport per unit area would no longer be predictable with this model. This region on each slice is dashed.

CYLINDRICAL COLUMNS

Basic Equations

As in the vertical-plate system, laminar Newtonian flow is assumed to prevail in vapor and liquid streams. The flow profiles can be computed by elementary classical methods and hence all other hydraulic characteristics of the column can be found.

The cylinder radius is x_0 and the radius of the vapor column x_s. The reflux liquid is assumed to flow downward under force of gravity as films on the walls. The liquid stratum is annular in form with width equal to $x_0 - x_s$.

The liquid and vapor are assumed, again, to be Newtonian; that is, the tractive force per unit area is assumed to be equal to the product of viscosity, η, a constant, and the shear rate dW/dx.

The elemental strip considered in this discussion is an annulus with an area of $2\pi x \Delta x$. Basic equations are developed from considerations of equilibrium of forces. As in the vertical-plate case, equilibrium conditions in the liquid layer are expressed by

$$\frac{d}{dx}\left(\eta_l x \frac{dW}{dx}\right) + xQ = 0 \qquad (1')$$

This equation is analogous to Equation (1) in the vertical-plate case but since the elemental strip is an annulus the form of the equation is necessarily different.

The corresponding vapor equation, with the assumption of negligible density, is

$$\frac{d}{dx}\left(\eta_v x \frac{dW_v}{dx}\right) + x\frac{dp}{dz} = 0 \qquad (2')$$

Integrating Equation (1′) twice, one gets

$$\eta_l W_l = \frac{-x^2 Q}{4} + C_1 \ln x + C_2 \qquad (3')$$

The corresponding vapor equation becomes after the first integration

$$\eta_v x \frac{dW_v}{dx} = \frac{-x^2}{2}\frac{dp}{dz} + C_3 \qquad (4')$$

and a second integration yields

$$\eta_v W_v = \frac{-x^2}{4}\frac{dp}{dz} + C_3 \ln x + C_4 \qquad (5')$$

Boundary Conditions

In the vapor stream, since the velocity profile is symmetric, $dW_v/dx = 0$ at $x = 0$. Therefore, Equation (4′) yields $C_3 = 0$, and so Equation (5′) becomes

$$\eta_v W_v = \frac{-x^2}{4}\frac{dp}{dz} + C_4 \qquad (5'a)$$

In the liquid layer $W_l = 0$ for $x = x_0$ (no slip condition at walls); hence Equation (3′) gives

$$C_2 = \frac{x_0^2 Q}{2} - C_1 \ln x_0$$

and so Equation (3′) can be written as

$$\eta_l W_l = \frac{Q}{4}(x_0^2 - x^2) + C_1 \ln x/x_0 \qquad (6')$$

Further, since W_l is finite for any x such that $0 \leq x \leq x_0$, C_1 must be 0 and Equation (6′) for velocity in the liquid layer becomes

$$\eta_l W_l = \frac{Q}{4}(x_0^2 - x^2) \qquad (6'a)$$

This corresponds to Equation (6) in the vertical-plate case.

In the vapor stream, since the limiting velocities at the interface of the liquid and vapor streams are assumed equal, one can write $W_v = W_l$ at $x = x_s$ so that Equation (6′a) may be set equal to (5′a) when $x = x_s$, whence a solution for C_4 gives

$$C_4 = \frac{x_s^2}{4}\frac{dp}{dz} + \frac{\eta_v}{4\eta_l} Q(x_0^2 - x_s^2)$$

Finally the expression for vapor becomes

$$\eta_v W_v = \frac{1}{4}\frac{\eta_v}{\eta_l} Q(x_0^2 - x_s^2) + \frac{dp}{dz}(x_s^2 - x^2) \qquad (7')$$

which is similar to Equation (7) in the vertical-plate case.

This completes the velocity-profile developments. Equations (6′a) and (7′) are quite essential in all that follows.

Expressions for Pressure Gradient and Mass Transport

Here, as in the vertical plate, explicit expressions will be obtained for the pressure gradient and mass transport by merely assuming an equilibrium state. Both the vapor mass transport and liquid mass transport per unit time are obtained by merely integrating over the velocity profiles. Then under assumption of equilibrium these two are set equal and the resulting equality yields an expression for pressure gradient.

Since

$$M_l = 2\pi\rho_l \int_{x_s}^{x_0} W_l x\,dx$$

$$= \frac{2\pi}{4}\frac{\rho_l}{\eta_l} Q \int_{x_s}^{x_0} x(x_0^2 - x^2)\,dx$$

$$= \frac{\pi}{2}\frac{\rho_l}{\eta_l} Q\left(\frac{x_0^2 - x_s^2}{2}\right)^2 \qquad (8')$$

and

$$M_v = 2\pi\rho_v \int_0^{x_s} W_v x\,dx$$

$$= \frac{2\pi\rho_v}{4\eta_v}\int_0^{x_s}\left[\frac{dp}{dz}(x_s^2 x - x^3) + \frac{\eta_v}{\eta_l} Q(x_0^2 - x_s^2)x\right]dx$$

$$= \frac{\pi\rho_v}{2\eta_v}\left[\frac{x_s^4}{4}\frac{dp}{dz} + \frac{Q}{2}\frac{\eta_v}{\eta_l}(x_0^2 - x_s^2)x_s^2\right] \qquad (9')$$

Then setting $M_v = -M_l$ and manipulating, one gets

$$\frac{dp}{dz} = -\rho_l g \frac{(1-u^2)^2 + k_1(1-u^2)u^2}{(1-u^2)^2 + k_2 u^4 + 2k_1 u^2(1-u^2)} \qquad (10')$$

where $k_1 = \rho_v/\rho_l : k_2 = k_1\eta_l/\eta_v$ and $u = x_s/x_0$.

Here, too, dp/dz is monotonic, varying from 0 to $-\rho_l g$ for $1 \geq u \geq 0$.

For calculation purposes it is better to write Equation (9′) as

$$M_v = \frac{\pi\rho_v x_0^4 u^2}{8\eta_v} \cdot \left[\left(\frac{2\eta_v Q}{\eta_l} - \frac{dp}{dz}\right)u^2 + \frac{2\eta_v Q}{\eta_l}\right] \qquad (10'a)$$

whence the average mass transport per (centimeter square^{-1})(second^{-1}), is

$$\frac{M_v}{\pi x_s^2} = \frac{\rho_v x_0^2}{8\eta_v} \cdot \left[\left(\frac{2\eta_v Q}{\eta_l} - \frac{dp}{dz}\right)u^2 + \frac{2\eta_v Q}{\eta_l}\right] \qquad (10'b)$$

Expression (10′b) has the value 0 at $u = 0$ since $dp/dz = -\rho_l g$ at $u = 0$. It has the value 0 at $u = 1$ since $dp/dz = 0$ at $u = 1$. Since the function is positive throughout $0 \leq u \leq 1$, it has a maximum for any assigned value of x_0.

Critical Values of Radius

As in the vertical-plate case, if one uses the Reynolds number to determine a critical velocity, certain limitations on the values of u and x_0 will exist when this critical velocity is interpreted as the onset of turbulence. Values of the velocity (average velocity) in excess of the critical velocity would lead to turbulence. The critical velocity is given by $W_{Re} = (Re\ \eta_v)/(2x_s\rho_v)$ where Re is Reynolds number and $2x_s$ is hydraulic diameter. The average velocity in the vapor stream is

$$\overline{W}_v = \frac{1}{x_s}\int_0^{x_s} W_v\,dx \qquad (11')$$

$$= \frac{1}{4\eta_v}\left[\frac{2}{3}x_s^2\frac{dp}{dz} + \frac{\eta_v Q}{\eta_l}(x_0^2 - x_s^2)\right]$$

If one assumes nonturbulent flow, then $\overline{W}_v \leq W_{Re}$; hence using (11′) and the expression for W_{Re}, one gets as an inequality for x_0

$$x_0^3 \leq \frac{6\eta_v\eta_l Re}{\rho_v\left[2\frac{k_2}{k_1}\frac{dp}{dz}u^3 + 3Qu(1-u^2)\right]} \qquad (12')$$

This again merely states that for a given u, since dp/dz can be determined from (10′), the largest x_0 is given by the equality in (12′). If x_0 satisfies the inequality for a fixed u, the mass transport per unit area is not at maximum.

If the equality in (12′) were looked upon as an equation relating x_0 and u, it is obvious that

$$x_0^3 \geq \frac{6\eta_v\eta_l Re}{\rho_v \max\left[2\frac{k_2}{k_1}\frac{dp}{dz}u^3 + 3Qu(1-u^2)\right]} \qquad (13')$$

This is interpreted as meaning that x_0 has a smallest value corresponding to the value of u which maximizes the denominator. Such a value of x_0 and corresponding value of u can be interpreted as the best values of the design parameters for maximizing the mass transport per unit area. (See Table 1.)

No simple expression analogous to Equation (14) in the vertical-plate case results in this case. Table 1′ shows that vapor mass transport (grams/second) decreases with decreasing x_0 and the mass transport per unit area increases up to a maximum.

Example

$\rho_l = 0.66$ g./ml. $\quad \eta_l = 0.0027$ poise

$\rho_v = 0.0036$ g./ml. $\quad \eta_v = 0.00011$ poise

Re of the vapor = 2,100

$g = 980.65$ dynes/sq. cm.

In Table 1′ dp/dz was calculated for selected values of u by use of Equation (10′) and x_0 was calculated by use of Equation (12′). In the calculation of x_0 from (12′) the denominator of the right member had a maximum in the range of $u = 0.80$ to 0.79. This value then determines the optimum value for x_0 from Equation (13′). The last column shows that the optimum value for mass transport per unit area results for an $x_0 = 0.0451$ cm.

The interpretation of these results is identical to interpretation in the vertical-plate case, and Figure 1 is again applicable in a general sense.

TABLE 1′

u	$1/(\rho_l g)\ dp/dz$	x_0	M_v, g./sec.	M_v, g./(sq. cm.)(sec.)
0.95	−0.0876	0.0776	1.933×10^{-2}	1.132
0.90	−0.3008	0.0537	1.305×10^{-2}	1.778
0.85	−0.5312	0.0468	1.074×10^{-2}	2.160
0.80	−0.7007	0.0451	0.971×10^{-2}	2.374
0.79	−0.7340	0.0451	0.938×10^{-2}	2.354
0.75	−0.8209			

NOTATION

C_1, C_2, C_3, C_4 = arbitrary constants in differential equations
D = hydraulic diameter
dp/dz = vertical pressure gradient
dW/dx = shear rate
g = gravitational constant
H_l = liquid holdup
k_1 = ρ_v/ρ_l
k_2 = $k_1\eta_l/\eta_v$
M_l = mass liquid flow, g./sec.
M_v = mass vapor flow, g./sec.
Q = $dp/dz + \rho_l g$
Re = Reynolds number (in vapor stream)
u = x_s/x_0
W_l = linear velocity in liquid
W_v = linear velocity in vapor
$\overline{W}_v$ = average linear velocity in vapor
x_0 = half-plate distance, or radius of cylinder
x_s = half-vapor width, or radius of vapor stream
Δ = incremental operator
η_l = viscosity of liquid
η_v = viscosity of vapor
ρ_l = density of liquid
ρ_v = density of vapor

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